2.20 Iterative Methods

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Name: **Date:** **Score:**

1. Using $x_0 = 1$ and $x_{n+1} = 2x + 1$, find x_4

/2

2. Using $x_0 = 4$ and $x_{n+1} = \frac{x}{4} + 1$, find x_3

/2

3. Using $x_0 = 3$ and $x_{n+1} = \sqrt{x} + 2$, find x_1, x_2 and x_3

/3

4. The number of rabbits in a field t days from now is defined by $N_0 = 210$ and $N_{t+1} = 1.15(N_t - 10)$. How many rabbits will be in the garden 3 days from now?

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5. The population in a town t years from now is defined by $N_0 = 55{,}000$ and $N_{t+1} = 1.09(N_t - 3700)$. How many people will be living in the town 4 years from now?

/3

6. Using $x_0 = 3$ and $x_{n+1} = 3 + \frac{9}{x_n}$, find x_1, x_2 and x_3

/3

7. Show that $x^3 - 8x - 10 = 0$ has a solution between 3 and 4

/1

8. Show that $x^3 + 2x = 1$ has a solution between 0 and 1

/1

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9. Show that $x^3 + 2x = 1$ can be rearranged to give $x = \frac{1}{2} - \frac{x^3}{2}$

/2

10. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{2} - \frac{x^3}{2}$ twice to estimate the solution to $x^3 + 2x = 1$

/2

11. The value of a car today is £17,000. Given that the value of the car changes according to by $V_{t+1} = 0.8(V_t - 500)$, and $t = time\ in\ years$, find the value of the car in 3 years.

/3

12. The number of bacteria in a colony change according to $N_{t+1} = 1.9(N_t - 200)$, where $t = time\ in\ hours$. If there are initially 4,000 bacteria in a colony, how many will there be in 8 hours?

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13. A solution to the equation $x^3 - 3x - 1 = 0$ lies between -1 and -2. By considering values in this interval, find a solution to this equation to 1dp.

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14. A solution to the equation $3x^3 + 6x - 4 = 0$ lies between 0 and 1. By considering values in this interval, find a solution to this equation to 1dp.

/2

15. A solution to the equation $x^3 - 2x^2 + 19 = 0$ lies between -3 and -2. By considering values in this interval, find a solution to this equation to 1dp.

/2

16. Using $x_0 = 4.5$ and $x_{n+1} = \sqrt{2x} + 2$, find x_1, x_2 and x_3

/3

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17. Show that the equation $x^4 - 5x + 1 = 0$ has a root between 1.5 and 2

/2

18. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{x_n+3}$ three times to find an estimate for the solution to $x^3 + 3x = 2$

/3

19. Using $x_0 = 1$ and $x_{n+1} = 2x + \frac{3x}{2} + 1$, find x_4

/2

20. Invent your own question and see if a parent/guardian can answer it

/0

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Answers

1. Using $x_0 = 1$ and $x_{n+1} = 2x + 1$, find x_4

$x_1 = 2x_0 + 1$

$x_1 = 2 \times 1 + 1$

$x_1 = 3$ [0.5]

$x_2 = 2 \times 3 + 1$

$x_2 = 7$ [0.5]

$x_3 = 2 \times 7 + 1$

$x_1 = 15$ [0.5]

$x_4 = 2 \times 15 + 1$

$x_4 = 31$ [0.5]

/2

2. Using $x_0 = 4$ and $x_{n+1} = \frac{x}{4} + 1$, find x_3

$x_1 = \frac{x_0}{4} + 1$

$x_1 = \frac{4}{4} + 1$

$x_1 = 2$ [1]

$x_2 = \frac{2}{4} + 1$

$x_2 = 1.5$

$x_3 = \frac{1.5}{4} + 1$

$x_3 = 1.375$ [1]

/2

3. Using $x_0 = 3$ and $x_{n+1} = \sqrt{x} + 2$, find x_1, x_2 and x_3

$x_1 = \sqrt{x_0} + 2$

$x_1 = \sqrt{3} + 2$

$x_1 = 3.732..$ [1]

$x_2 = \sqrt{3.732} + 2$

$x_2 = 3.9318 \ldots$ [1]

$x_3 = \sqrt{3.9318..} + 2$

$x_3 = 3.9828 \ldots$ [1]

/3

4. The number of rabbits in a field t days from now is defined by $N_0 = 210$ and $N_{t+1} = 1.15(N_t - 10)$. How many rabbits will be in the garden 3 days from now?

$N_1 = 1.15(N_0 - 10)$

$N_1 = 1.15(210 - 10)$

$N_1 = 230$ [1]

$N_2 = 1.15(230 - 10)$

$N_2 = 253$ [1]

$N_3 = 1.15(253 - 10)$

$N_3 = 279.45$ [1]

So there should be 279 rabbits in 3rd day.

/3

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Answers

5. The population in a town t years from now is defined by $N_0 = 55{,}000$ and $N_{t+1} = 1.09(N_t - 3700)$. How many people will be living in the town 4 years from now?

$N_1 = 1.09(N_0 - 3700)$

$N_1 = 1.09(55{,}000 - 3700)$

$N_1 = 55{,}917$ [1]

$N_2 = 1.09(55{,}917 - 3700)$

$N_2 = 56{,}916.53$

$N_3 = 1.09(56{,}916.53 - 3700)$

$N_3 = 58{,}006.0177$ [1]

$N_4 = 1.09(58{,}006.0177 - 3700)$

$N_4 = 59193.559$ [1]

so there should be 59193 *people after* 4 *years.*

/3

6. Using $x_0 = 3$ and $x_{n+1} = 3 + \frac{9}{x_n}$, find x_1, x_2 and x_3

$x_1 = 3 + \frac{9}{x_0}$

$x_1 = 3 + \frac{9}{3}$

$x_1 = 6$ [1]

$x_2 = 3 + \frac{9}{6}$

$x_2 = 4.5$ [1]

$x_3 = 3 + \frac{9}{4.5}$

$x_3 = 5$ [1]

/3

7. Show that $x^3 - 8x - 10 = 0$ has a solution between 3 and 4

let's substitute $x = 3$ in $x^3 - 8x - 10$

$3^3 - 8 \times 3 - 10 = -7$

Then substitute $x = 4$ in $x^3 - 8x - 10$

$4^3 - 8 \times 4 - 10 = 22$

So here there is a change of signs so $x^3 - 8x - 10$ should have a solution between 3 and 4. [1]

/1

8. Show that $x^3 + 2x = 1$ has a solution between 0 and 1

let's substitute $x = 0$ in $x^3 + 2x - 1$

$0^3 + 2 \times 0 - 1 = -1$

Then substitute $x = 1$ in $x^3 + 2x - 1$

$1^3 + 2 \times 1 - 1 = 2$

So here there is a change of signs so $x^3 + 2x - 1$ should have a solution between 0 and 1. [1]

/1

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Answers

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9. Show that $x^3 + 2x = 1$ can be rearranged to give $x = \frac{1}{2} - \frac{x^3}{2}$

$$x^3 + 2x = 1$$

$$2x = 1 - x^3 \quad [1]$$

$$x = \frac{1 - x^3}{2}$$

$$x = \frac{1}{2} - \frac{x^3}{2} \quad [1]$$

/2

10. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{2} - \frac{x^3}{2}$ twice to estimate the solution to $x^3 + 2x = 1$

$$x_{n+1} = \frac{1}{2} - \frac{x^3}{2}$$

$x_1 = \frac{1}{2} - \frac{0^3}{2}$

$x_1 = 0.5$ [1]

$x_2 = \frac{1}{2} - \frac{0.5^3}{2}$

$x_2 = 0.4375$ [1]

So solution to $x^3 + 2x = 1$ *is* $x = 0.4375$

/2

11. The value of a car today is £17,000. Given that the value of the car changes according to by $V_{t+1} = 0.8(V_t - 500)$, and $t = \text{time in years}$, find the value of the car in 3 years.

$V_1 = 0.8(V_{t0} - 500)$

$V_1 = 0.8(17000 - 500)$

$V_1 = 13{,}200$ [1]

$V_2 = 0.8(13{,}200 - 500)$

$V_2 = 10{,}160$ [1]

$V_3 = 0.8(10{,}160 - 500)$

$V_3 = 7728$

So value of the car after 3 years is £7728 [1]

/3

12. The number of bacteria in a colony change according to $N_{t+1} = 1.9(N_t - 200)$, where $t = \text{time in hours}$. If there are initially 4,000 bacteria in a colony, how many will there be in 8 hours?

$N_1 = 1.9(N_0 - 200)$

$N_1 = 1.9(4000 - 200)$

$N_1 = 7220$

$N_2 = 1.9(7220 - 200)$

$N_2 = 13{,}338$

$N_3 = 1.9(13{,}338 - 200)$

$N_3 = 24962.2$

$N_4 = 1.9(24962.2 - 200)$

$N_4 = 47048.18$

$N_5 = 1.9(47048.18 - 200)$

$N_5 = 89011.542$

$N_6 = 1.9(89011.542 - 200)$

$N_6 = 168741.9298$

$N_7 = 1.9(168741.93 - 200)$

$N_7 = 320229.67$

$N_8 = 1.9(320229.67 - 200)$

$N_8 = 608056.3666$

/3

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Answers

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13. A solution to the equation $x^3 - 3x - 1 = 0$ lies between -1 and -2. By considering values in this interval, find a solution to this equation to 1dp.

$x^3 - 3x - 1 = 0$

Substitute $x = -1.5$

$-1.5^3 - 3 \times -1.5 - 1 = 0.125$ too big

Substitute $x = -1.6$

$-1.6^3 - 3 \times -1.6 - 1 = -0.296$ too small

So answer should be in between -1.5 and -1.6 [1]

Substitute -1.54, -1.55 and -1.56 then you can see answer is closer to -1.5 than -1.6.

So answer should be -1.5 [1]

/2

14. A solution to the equation $3x^3 + 6x - 4 = 0$ lies between 0 and 1. By considering values in this interval, find a solution to this equation to 1dp.

$3x^3 + 6x - 4 = 0$

Substitute $x = 0.5$

$3 \times 0.5^3 + 6 \times 0.5 - 4 = -0.625$ too small [1]

Substitute $x = 0.6$

$3 \times 0.6^3 + 6 \times 0.6 - 4 = 0.24$ close, but too

Substitute $x = 0.7$

$3 \times 0.7^3 + 6 \times 0.7 - 4 = 1.22, far$ too big

Answer should be 0.6 since there is a change of signs between 0.5 and 0.6, and 0.24 is closer to 0 than -0.625 [1]

/2

15. A solution to the equation $x^3 - 2x^2 + 19 = 0$ lies between -3 and -2. By considering values in this interval, find a solution to this equation to 1dp.

$x^3 - 2x^2 + 19 = 0$

Substitute $x = -2.5$

$-2.5^3 - 2 \times -2.5^2 + 19 = -9.125$ too small [1]

Substitute $x = -2.3$

$-2.3^3 - 2 \times -2.3^2 + 19 = -3.747$ too small

Substitute $x = -2.2$

$-2.2^3 - 2 \times -2.2^2 + 19 = -1.328$ close, but still too small

Substitute $x = -2.1$

$-2.1^3 - 2 \times -2.1^2 + 19 = 1.081$ too big

Answer should be -2.1 since there is a change of signs between -2.1 and -2.2 and 1.081 is closer to 0 than -1.328 [1]

/2

16. Using $x_0 = 4.5$ and $x_{n+1} = \sqrt{2x} + 2$, find x_1, x_2 and x_3

$x_1 = \sqrt{2x_0} + 2$

$x_1 = \sqrt{2 \times 4.5} + 2$

$x_1 = 5$ [1]

$x_2 = \sqrt{2 \times 5} + 2$

$x_2 = 5.162$ ...[1]

$x_3 = \sqrt{2 \times 5.162..} + 2$

$x_3 = 5.2131$ ...[1]

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Answers

17. Show that the equation $x^4 - 5x + 1 = 0$ has a root between 1.5 and 2

let's substitute $x = 1.5$ in $x^4 - 5x + 1$

$1.5^4 - 5 \times 1.5 + 1 = -1.4375$

Then substitute $x = 2$ in $x^4 - 5x + 1$

$2^4 - 5 \times 2 + 1 = 7$

So here there is a change of signs so $x^4 - 5x + 1$ should have a solution between 1.5 and 2. [1]

/2

18. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{x_n+3}$ three times to find an estimate for the solution to $x^3 + 3x = 2$

$x_1 = \frac{2}{x_0 + 3}$

$x_1 = \frac{2}{0+3}$

$x_1 = \frac{2}{3}$ [1]

$x_2 = \frac{2}{\frac{2}{3} + 3}$

$x_2 = 0.5454 ....$ [1]

$x_3 = \frac{2}{0.5454 ... + 3}$

$x_3 = 0.5641 ...$

$So\ answer = 0.5641$ [1]

/3

19. Using $x_0 = 1$ and $x_{n+1} = 2x + \frac{3x}{2} + 1$, find x_4

$x_1 = 2x + \frac{3x_0}{2} + 1$

$x_1 = 2 \times 1 + \frac{3 \times 1}{2} + 1$

$x_1 = 4.5$ [0.5]

$x_2 = 2 \times 4.5 + \frac{3 \times 4.5}{2} + 1$

$x_2 = 16.75$ [0.5]

$x_3 = 2 \times 16.75 + \frac{3 \times 16.75}{2} + 1$

$x_3 = 59.625$ [0.5]

$x_4 = 2 \times 59.625 + \frac{3 \times 59.625}{2} + 1$

$x_4 = 209.6875$ [0.5]

/2

20. Invent your own question and see if a parent/guardian can answer it

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